Astroparticle Physics
Graded Exercise 1

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This exercise will be graded and must be uploaded to your personal Dropbox folder ***before 17:15*** *(!). This is part of the examination of this course and has to be done individually. Please do not communicate with the other students, or use any other type of resources (offline or online). Because of this, it is important that you have uploaded a signed student declaration in your Dropbox folder.*

Total number of points: $\left(\frac{\Sigma \textbf{points}}{7.5} \times 9\right) + 1$**. There are 7.5 total points**

Question 1

a. Explain in your own words the concept of cross section, and what units are used to measure it. **0.5pt**

Question 2

Consider the process

$$\gamma \to e^- + e^+$$

known as pair production. Pair production must happen in the Coulomb field of an atomic nucleus, to ensure that energy and momentum are conserved. Due to this the nucleus receives some recoil, and hence momentum. Assume the nucleus is initially at rest. Conservation of four momentum requires that

$$q_\gamma + q_{\text{nuc}} = q_{e^+} + q_{e^-} + q'_{\text{nuc}}$$

where q_{nuc} is the four momentum of the (at rest) nucleus *before* the process and q'_{nuc} is the four momentum of the nucleus *after* the process.

a. Calculate the four momentum in the lab frame before the process (q_{lab}) and after the process (q'_{lab}). **2pt**

b. Compute the four momenta in the COM frame after the process (q'_{COM}). *Hint: What does the COM frame imply about the three momentum?* **1pt**

c. Following the proposed steps, demonstrate that without the presence of the nucleus to absorb excess momentum, the pair production process cannot occur (i.e. is forbidden).

   (i) Derive a relation between q_{lab} and q'_{lab}, using the fact that the four momentum squared is conserved. Assume that the energies of the electron and positron are equal.
   Hint: Remember not to include the nucleus/ recoil. $\vec{v} \cdot \vec{u} = vu\cos\theta$ *might be useful. Equal energies implies that the magnitude of momenta are equal.* **1pt**

(ii) Express the result of (i) as a function of the electron/ positron mass m_e, the Lorentz factor γ, and the angle θ between the electron and positron emission. *Hint: Use the energy-momentum relation* ($E^2 = p^2 + m^2$) *and the energy-mass relation* ($E = \gamma m$), *and then factorize out* m_e. **1pt**

(iii) Argue that the expression found in (ii) is not valid, and hence pair production does not work without the presence of a nucleus. **2 pt**

Some Useful Equations

The total energy E, the kinetic energy E_k and the particle momentum p are all related as follows

$$\beta = \frac{v}{c} \tag{1}$$

$$p = \gamma m_0 v = \beta\gamma m_0 c \tag{2}$$

$$E = \gamma m_0 c^2 \tag{3}$$

$$E^2 = p^2c^2 + m_0^2c^4 \tag{4}$$

$$E = E_K + m_0c^2 \rightarrow E_K = E - m_0c^2 \tag{5}$$

$$\beta = \frac{pc}{\sqrt{p^2c^2 + m_0^2c^4}} \tag{6}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \sqrt{1 + \frac{p^2c^2}{m_0^2c^4}} = \frac{E}{m_0c^2} = 1 + \frac{E_k}{m_0c^2} \tag{7}$$

$$\beta\gamma = \frac{pc}{m_0c} \tag{8}$$

m_0 is the rest mass of the particle in question.